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Journal of Power Sources 158 (2006) 245–253
JOURNAL OF POWER SOURCES
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Predictive control of solid oxide fuel cells using fuzzy Hammerstein models

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Received 1 March 2005; received in revised form 16 July 2005; accepted 16 August 2005
Available online 18 October 2005

Abstract

This paper presents a study of fuzzy Hammerstein models as part of a model predictive control strategy. The model configuration includes a nonlinear static block followed by a linear dynamic block, where the static nonlinearity is represented by a fuzzy model.

The model and controller have been realized in the MATLAB environment. Simulation examples demonstrate the potential of such structured models for application to the control of a solid oxide fuel cell.

Keywords: Fuzzy modeling; Hammerstein systems; Model predictive control; Solid oxide fuel cells

1. Introduction

Solid oxide fuel cell (SOFC) has attracted considerable interest during the past decade as highly effective and environmentally acceptable sources of electrical energy. The SOFC power plant is known to be a potential alternative in the electric utility, for domestic, commercial and industrial sectors. It produces less harmful chemical and acoustic emissions at higher efficiency than the conventional technologies.

The main features of the SOFC are all solid-state construction and high-temperature operation. The combination of these features leads to a number of unique characteristics and advantages for this type of fuel cell, including flexibility in cell and stack designs, manufacturing processes, and power plant sizes.

Padullés et al. [1] develop a SOFC model, which includes species dynamics, but it does not consider temperature dynamics. Hall and Colclaser [2] modeled a 3-kW SOFC but they did not take into account dynamics of the chemical species. Achenbach develops a mathematical model of a planar SOFC, which concentrates on effects of temperature changes on output voltage response [3]. Temperature dynamics is modeled in a three-dimensional (3-D) vector space. The same author investigated the transient behavior of a stand-alone SOFC caused by a load change in [4]. It shows that the relaxation time of the output voltage is highly related to the effect of temperature dynamics. The application of the *AutoRegression with eXogenous signal* (ARX) identification algorithm to compute linear system models is presented in [5]. The *Nonlinear AutoRegressive Exogenous* (NARX) approach is used in [6] to analyze the dynamics of this fuel cell.

For nonlinear dynamic systems, the conventional techniques of modeling and identifications are difficult to implement and sometimes impracticable. However, others techniques based on fuzzy logic are more and more used for modeling this kind of process [7]. Among the different fuzzy methods, the Takagi–Sugeno model has attracted most attention [8]. In fact, this model consists of if-then rules with fuzzy antecedents and mathematical functions in the consequent part. The task of system identification is to determine both the nonlinear parameters of the antecedents and the linear parameters of the rules consequent. A fuzzy logic control of three-phase inverters for fuel cell systems is presented in [9].

Takagi–Sugeno fuzzy models are suitable to model a large class of nonlinear systems [10,11]. Fuzzy modeling and identification from measured data are effective tools for the approximation of uncertain nonlinear systems. Most attention has been devoted to *single-input*, *single-output* (SISO) or *multi-input*, *single-output* (MISO) systems. Recently, also methods have

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doi:10.1016/j.jpowsour.2005.08.041

Nomenclature

C_p^s	heat capacity of the cell unit (J kg^{-1} K^{-1})
E_0	standard reversible cell potential (V)
F	Faraday's constant (C kmol^{-1})
$\bar{h}_{ai}^{in}$, $\bar{h}_{ci}^{in}$	anode (cathode) inlet partial molar enthalpies (J mol^{-1})
$\bar{h}_i^s$	partial molar enthalpies at stack temperature (J mol^{-1})
i	stack current density (A m^{-2})
i_L	limiting current density (A m^{-2})
i_0	exchange current density (A m^{-2})
I	stack current (A)
K_i	valve molar constants
K_r	constant dependent on Faraday's constant and number of electrons in the reaction (kmol s$^-$ A^{-1})
M^s	mass of the cell unit (kg)
n	number of electrons participating in the reaction
N_i^{in}, N_i^o	molar flow rates (mol s^{-1}) of the ith reactant at the cell input and output, respectively
N_i^r	reaction rate (mol s^{-1}) of the ith reactant
N_{H_2}	hydrogen flow that reacts (kmol s^{-1})
N_0	number of cells in stack
N_{ta}^{in}, N_{tc}^{in}	anode (cathode) total inlet molar flow (mol s^{-1})
p_{H_2}, p_{H_2O}	partial pressures of hydrogen and water (atm)
p_i	partial pressure of the ith reactant (atm)
P_{dc}	stack dc power (W)
r	ohmic resistance (Ω m^2)
R	gas constant (8.31 J mol^{-1} K^{-1})
R_{ai}, R_{ci}	anode (cathode) total rate of production of species (mol s^{-1})
T^s	stack solid average temperature (K)
T^0	temperature constant (K)
V	compartment volume (m^3)
V_{dc}	cell voltage (V)
V_e	volume of the cell unit (m^3)
V_o	open-circuit reversible potential (V)
x_i	mole fractions of species
x_{ai}^{in}, x_{ci}^{in}	anode (cathode) inlet mole fractions

Greek letters

α	electron transfer coefficient of the reaction at the electrode
α_r, β_r	ohmic resistance constants
β	scaling factor
η_{act}	activation losses (V)
η_{con}	concentration losses (V)
τ_{H_2}	time constant associated with the hydrogen flow and is a function of temperature (s)
ξ	total gas components in anode or cathode

been proposed to deal with *multi-input*, *multi-output* (MIMO) systems [12–14].

The Hammerstein models are special kinds of nonlinear systems where the nonlinear block is static and is followed by a linear system. These models have applications in many engineering problems and therefore, identification of Hammerstein models has been an active research area for many years. There exist a large number of research papers in the literature on the topics of Hammerstein model identifications [15–18].

In this paper, a fuzzy Hammerstein (FH) model to represent SOFC is introduced where the static nonlinearity is represented by a fuzzy model. Therefore, the model can be identified with the help of input–output data.

Model predictive control (MPC) has been an active field of research during the last three decades, driven both by numerous successful applications of the technology [19–21] and by the research interests of the academia. The main reason of this success is the ability of MPC to control multivariable systems under constraints in an optimal way. In model predictive control, the control action is computed by solving an optimization problem on line in each sampling period. This is the main difference from conventional control, where a precomputed control law is employed. Many applications of MPC based on fuzzy prediction models have been reported [22–25].

The output of the SOFC has a high voltage fluctuation in response to the load variations. On the other hand, the SOFC produces an unregulated voltage due to its internal dynamics. An MPC is used to control the output voltage of the SOFC by controlling the fuel flow to keep the voltage at a desired value.

The paper is organized as follows. In Section 2, general principles of SOFC are explained. In Section 3, the FH model is introduced. The MPC is formulated in Section 4. Simulation examples illustrating the performance of SOFC are presented in Section 5, and finally, conclusions are provided in Section 6.

2. Solid oxide fuel cell dynamic model

The proposed stack model is based on the following assumptions:

(1) Stack is fed with hydrogen and air, therefore the fuel processor dynamics is not included.
(2) A uniform gas distribution among cells is assumed, since there is a small deviation of the gas distribution among the cells.
(3) There is no heat transfer among cells. Each cell has the same temperature and current density [1].
(4) The channels that transport gases along the electrodes have a fixed volume, but their lengths are small, so that it is only necessary to define one single pressure value in their interior. The ratio of pressures between the interior and exterior of the channel is large enough to consider that orifice is choked [1,26].

Fig. 1 shows the dynamic model of SOFC, along with its major chemical reactions.

2.1. Electrochemical model

The change in concentration of each species that appears in the chemical reactions can be written in terms of input and out-

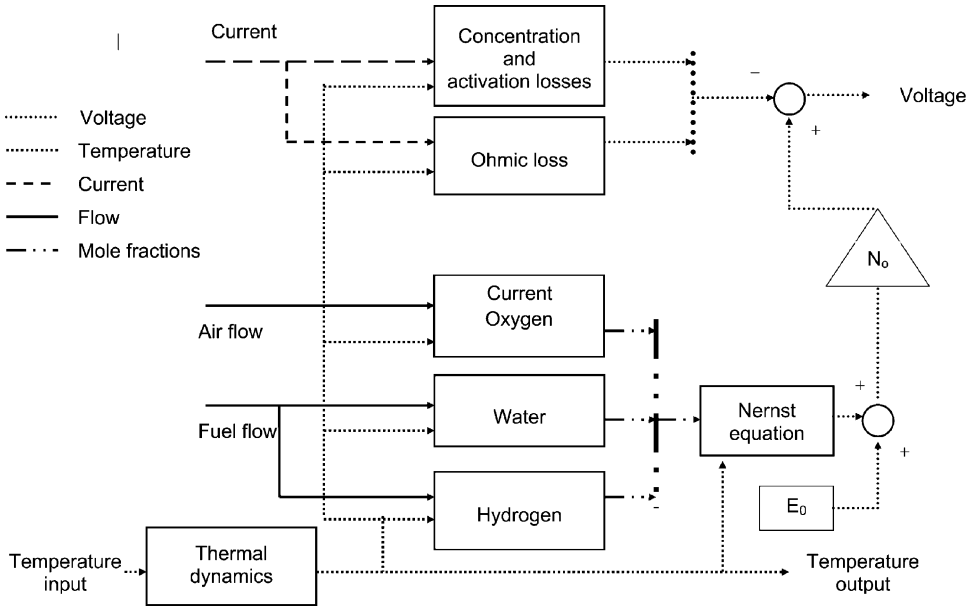

Fig. 1. Dynamic model of SOFC.

put flow rates and exit molarity due to the following chemical reaction [27,28]:

$$\frac{V}{RT}\frac{\mathrm{d}}{\mathrm{d}t}p_i = N_i^{\mathrm{in}} - N_i^{\mathrm{o}} - N_i^{\mathrm{r}} \tag{1}$$

In agreement with the basic electrochemical relationships, the molar flow that reacts can be calculated as:

$$N_i^{\mathrm{r}} = \frac{N_0 I}{2F} = 2K_{\mathrm{r}} I \tag{2}$$

For orifice that is choked [26], molar flow of any gas through the valve is proportional to its partial pressure inside the channel according to the following expressions [1]:

$$\frac{N_{\mathrm{H_2}}}{p_{\mathrm{H_2}}} = K_{\mathrm{H_2}}, \qquad \frac{N_{\mathrm{H_2O}}}{p_{\mathrm{H_2O}}} = K_{\mathrm{H_2O}} \tag{3}$$

Considering the hydrogen partial pressure,

$$\frac{V}{RT}\frac{\mathrm{d}}{\mathrm{d}t}p_{\mathrm{H_2}} = N_{\mathrm{H_2}}^{\mathrm{in}} - N_{\mathrm{H_2}}^{\mathrm{o}} - 2K_{\mathrm{r}} I \tag{4}$$

Applying the Laplace transformation to the above equations and isolating the hydrogen partial pressure, yields the following expressions:

$$p_{\mathrm{H_2}} = \frac{1/K_{\mathrm{H_2}}}{1 + \tau_{\mathrm{H_2}}(\mathrm{s})}(N_{\mathrm{H_2}}^{\mathrm{in}} - 2K_{\mathrm{r}} I) \tag{5}$$

$$\tau_{\mathrm{H_2}} = \frac{V}{K_{\mathrm{H_2}} RT} \tag{6}$$

A schematic of the system is displayed in Fig. 2.

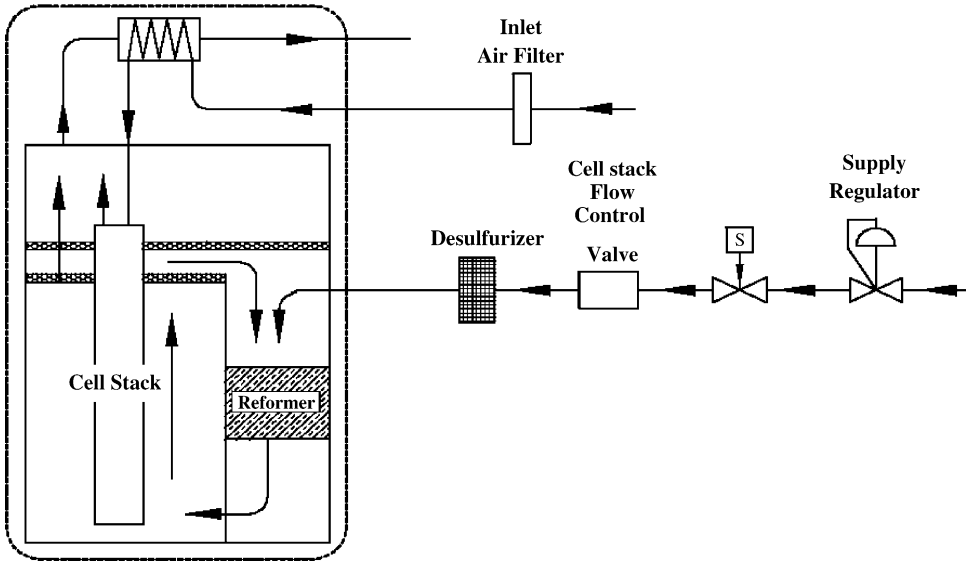

Fig. 2. Simplified flow schematic.

2.2. Thermal model

The fuel cell power output is closely related to the temperature of the cell unit. The heat storage in the thin fuel unit gas or oxidant gas layer is neglected. The thin fuel unit or oxidant gas layers are lumped to the cell unit and gas layers are assumed to have the same temperature as the cell unit [2–4].

The energy balance equation for each cell unit is as follows:

$$M^{s}C_{p}^{s}\frac{dT^{s}}{dt} = N_{ta}^{in}\left[\sum_{i=1}^{\xi}x_{ai}^{in}(\bar{h}_{ai}^{in}-\bar{h}_{i}^{s})\right]-\sum_{i=1}^{\xi}\bar{h}_{i}^{s}R_{ai} + N_{tc}^{in}\left[\sum_{i=1}^{\xi}x_{ci}^{in}(\bar{h}_{ci}^{in}-\bar{h}_{i}^{s})\right]-\sum_{i=1}^{\xi}\bar{h}_{i}^{s}R_{ci}-P_{dc} \quad (7)$$

Under the ideal gas supposition, the partial molar enthalpies are calculated using

$$\bar{h}_{i} = \bar{h}_{i}^{ref} + \int_{T_{ref}}^{T} c_{p,i}(u)\,du \quad (8)$$

and coefficients of the specific heats $c_{p,i}$,

$$c_{p,i} = a_i + b_i T + c_i T^2 + d_i T^3 \quad (9)$$

are encountered in standard reference tables. The reference enthalpy stands for energy at standard reference temperature and considers the heat of formation for each gas species to account for energy change on chemical reaction.

2.3. Nernst's equation

Applying Nernst's equation and Ohm's law (taking into account ohmic, concentration, and activation losses), the stack is connected in series and the stack output voltage is represented as follows [27–29]:

$$V_{dc} = V_{o} - rI - \eta_{act} - \eta_{con} \quad (10)$$

$$V_{o} = N_{0}\left(E_{0} + \frac{RT}{2F}\left[\ln\frac{x_{H_2}x_{O_2}^{0.5}}{x_{H_2O}}\right]\right) \quad (11)$$

For the reason that the reactant is consumed at the electrode by electrochemical reaction, there is a loss of potential due to the inability of the surrounding material to maintain the initial concentration of the bulk fluid. Concentration loss equation is given by [2,27–29]:

$$\eta_{con} = \frac{RT}{nF}\ln\left(1-\frac{i}{i_L}\right) \quad (12)$$

Activation polarization is existent when the rate of an electrochemical reaction at an electrode surface is controlled by sluggish electrode kinetics. Activation loss equation is as follows:

$$\eta_{act} = \frac{RT}{\alpha nF}\ln\left(\frac{i}{i_0}\right) \quad (13)$$

α is the transfer coefficient, which is considered to be the fraction of the change in polarization that leads to a change in the reaction rate constant and its value is usually 0.5 for the fuel cell application.

Tafel plots provide a visual understanding of the activation polarization of a fuel cell. They are used to measure the exchange current density, given by the extrapolated intercept at $\eta_{act} = 0$ which is a measure of the maximum current that can be extracted at negligible polarization, and the transfer coefficient (from the slope).

The usual form of the Tafel equation that can be easily expressed by a Tafel plot is,

$$\eta_{act} = a + b\ln i \quad (14)$$

where $\alpha = (-RT/\alpha nF)\ln i_0$ and $b = RT/\alpha nF$.

The term b is called the Tafel slope, and is obtained from the slope of a plot of η_{act} as a function of $\ln i$. The Tafel slope for an electrochemical reaction is about 110 mV per decade at room temperature. Decade is log current density (mA cm^{-2}).

Ohmic losses occur because of resistance to the flow of ions in the electrolyte and resistance to the flow of electrons through the electrode materials. This resistance is dependent on the cell temperature and is obtained by [29]:

$$r = \alpha_r \exp\left[\beta_r\left(\frac{1}{T^0}-\frac{1}{T}\right)\right] \quad (15)$$

3. Fuzzy Hammerstein model

Fuzzy systems can be used to model human knowledge in engineering problems. This knowledge may be classified into two categories: conscious knowledge explicitly expressible in words, and subconscious knowledge that a human expert transforms into actions but cannot explain in words.

Conscious knowledge can be expressed in terms of fuzzy if-then rules and implement the rules in fuzzy systems. For subconscious knowledge, the human expert can be considered as a black box. Thus, subconscious knowledge is represented by a set of input–output pairs. Hence, a problem of fundamental importance is to construct fuzzy systems from input–output pairs. To solve this problem, it is necessary to determine the membership functions of the fuzzy input and output sets, and define a fuzzy rule base.

Some of the most important applications of fuzzy theory have been focused on control problems where human operators, who intuitively know the behavior of the system, provide the best control. The human operator however may not always be able to satisfactorily control a process. Then, the control strategy may take the form of a set of situation–action pairs, known to the human operator, among which the control system can be designed. This provides the core knowledge for the system, but usually does not completely describe it. To perfect this description, one must define fuzzy sets representing the range of control parameters or sensor fields. The system description is then finely tuned by adjusting the fuzzy sets and the rule base.

The fuzzy Hammerstein model consists of a series connection of a memoryless nonlinearity, f, and linear dynamics, G, where

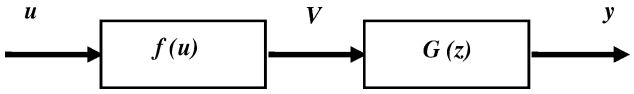

Fig. 3. Multivariable Hammerstein model.

$\boldsymbol{y} = [y_1, \ldots, y_{n_y}]^T$ is the output vector, $\boldsymbol{u} = [u_1, \ldots, u_{n_u}]^T$ the input vector, and $\boldsymbol{v} = [v_1, \ldots, v_{n_u}]^T$ represents the transformed input variables, as shown in Fig. 3 [30].

If the static nonlinearity is separately parameterized, $f(\cdot)$ can be formulated as a set of functions $v_h = f_h(\boldsymbol{u})$ for $h = 1, \ldots, n_u$. In this paper, the functions $f_h(\boldsymbol{u})$ are represented by zero-order Takagi–Sugeno fuzzy models formulated as a set of rules [8].

$$R_j^h : \text{If } u_1 \text{ is } A_{1,j} \quad \text{and} \quad u_{n_u} \text{ is } A_{n_u,j} \text{ then } v_h = p_j^h \tag{16}$$

From a given input vector, $\boldsymbol{u}$, the output of the fuzzy model, v_h, is inferred by computing the weighted average of the rule consequents:

$$v_h = \frac{\sum_{j=1}^{N_r} \beta_j(\boldsymbol{u}) p_j^h}{\sum_{j=1}^{N_r} \beta_j(\boldsymbol{u})} \tag{17}$$

The weight, $0 \le \beta_j(\boldsymbol{u}) \le 1$, represents the overall truth value of the jth rule calculated based on the degrees of membership.

The static nonlinearity is followed by a multivariable linear dynamic ARX model. Hence, the *Nonlinear AutoRegressive Moving Average with eXogenous input* (NAARX) model representation of the MIMO Hammerstein model is given by:

$$\hat{\boldsymbol{y}}(k) = \sum_{i=1}^{n_a} \mathbf{A}_i \boldsymbol{y}(k-i) + \sum_{i=1}^{n_b} \mathbf{B}_i f(\boldsymbol{u}(k-i-n_d)) \tag{18}$$

where $\boldsymbol{y}(k)$, …, $\boldsymbol{y}(k-n_a+1)$ and $\boldsymbol{u}(k-n_d)$, …, $\boldsymbol{u}(k-n_b-n_d+1)$ are the lagged outputs and inputs of the linear dynamic system, where n_a and n_b denote the maximum lags for the past outputs and inputs, and n_d is the discrete time delay. $\mathbf{A}_1, \ldots, \mathbf{A}_{n_a}$ and $\mathbf{B}_1, \ldots, \mathbf{B}_{n_a}$ are $n_y \times n_y$ and $n_y \times n_u$ matrices, respectively.

Hence, a compact form of the fuzzy Hammerstein model that represents a SISO process is formulated as,

$$\begin{aligned} \hat{\boldsymbol{y}}(k) &= \sum_{i=1}^{n_a} a_i y(k-1) + \sum_{i=1}^{n_b} b_i \sum_{j=1}^{N_r} \beta_j(u(k-i-n_d)) p_j \\ &= \sum_{i=1}^{n_a} a_i y(k-1) + \sum_{j=1}^{N_r} \sum_{i=1}^{n_b} b_i p_j \beta_j(u(k-i-n_d)) \end{aligned} \tag{19}$$

The parameters a_i and b_i belonging to the linear dynamic model are called the *linear parameters*, while the parameters p_j, belonging to the fuzzy model, are called the *nonlinear parameters*. The structure of the resulting model is shown in Fig. 4, where q denotes the shift operator, i.e., $u(k)q^{-1} = u(k-1)$.

4. Model predictive control

The first step in designing an MPC system is the derivation of a model that the controller will use for the optimization. This model should be as accurate as possible, while being simple enough to allow for repeated calculations during the optimization.

Model predictive control (MPC) refers to a class of control algorithms in which a dynamic process model is used to predict and optimize system performance.

MPC is rather a methodology than a single technique. The methodology of controllers belonging to the MPC family is characterized by the following strategy illustrated in Fig. 5.

As shown in Fig. 5, in MPC, the future outputs (fuel cell voltage) for a determined *prediction horizon* H_p are predicted at each instant k using a prediction model. These predicted outputs $\hat{\boldsymbol{y}}(k+j)$, $j = 1, \ldots, H_p$ depend on the state of the model at the

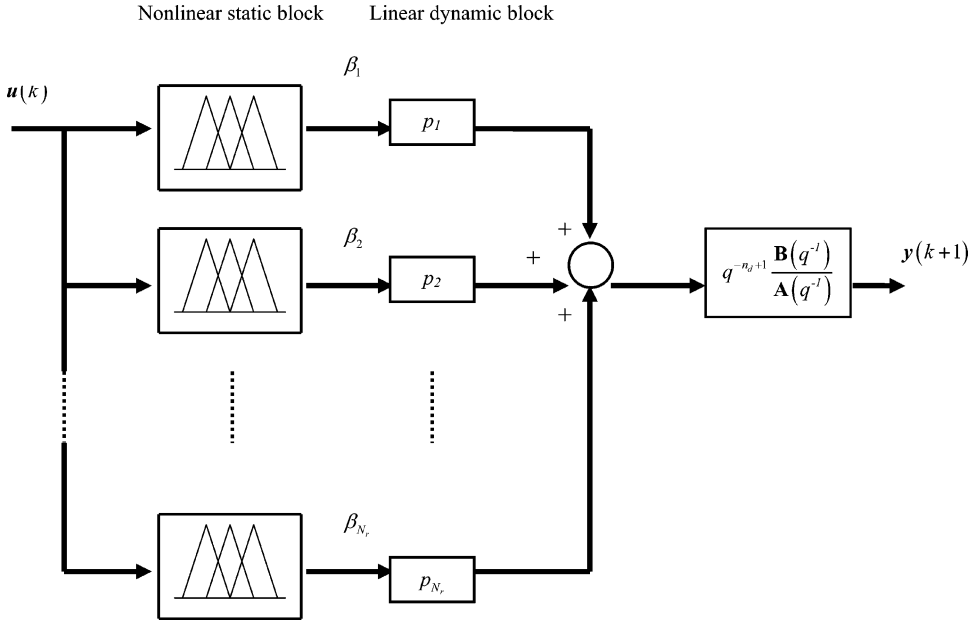

Fig. 4. Structure of the fuzzy Hammerstein model.

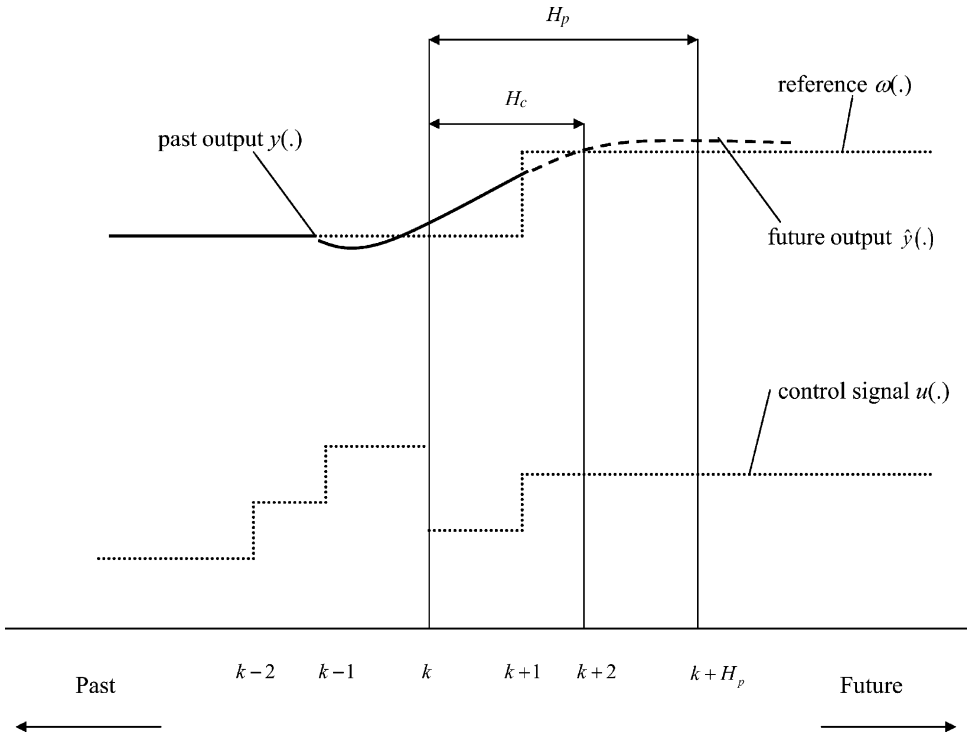

Fig. 5. Strategy of model predictive controller.

current time k (given, for instance, by the past inputs and outputs) and on the future control signals $u(k+j)$.

The control signal (fuel flow) changes only inside the *control horizon,* H_c, remaining constant afterwards,

$$u(k+j) = u(k+H_c-1), \quad j = H_c, ..., H_p \tag{20}$$

The set of control signals is calculated by optimizing a *cost function* in order to keep the process as close as possible to the *reference trajectory* (fuel cell voltage reference), $\omega(k+j)$, $j=1, \ldots, H_p$. This criterion usually requires a quadratic function of the errors between the predicted output signal and the reference trajectory. The control effort is included in the objective function in most cases. An explicit solution can be obtained if the criterion is quadratic, the model is linear and there are no constraints. Otherwise an iterative optimization method has to be used.

Due to the relatively simple block-oriented structure, the application of Hammerstein models in MPC is more straightforward than the application of the general NARX or NAARX models. In this section, the FH model is implemented in MPC by inverting the fuzzy model that represents static nonlinearity [31]. As the remaining part of the prediction model is the linear dynamic part of the FH model, the MPC optimization can be solved by quadratic programming.

The combination of the inverse fuzzy model and the nonlinear system results in a transformed dynamical system. This system is linear if the system is of the Hammerstein type and the static nonlinearity is identical to the fuzzy model.

As the inversion of the single-input single-output and multiple-input single-output fuzzy model is a straightforward analytical procedure, the computational demand of the controller is quite comparable to the linear generalized predictive control (GPC). This is a significant advantage compared to other nonlinear models, which require the use of nonlinear programming or linearization techniques.

In order to cope with the model-plant mismatch and also with disturbances (load changes), the internal model control (IMC) scheme [32] is used. The resulting scheme is depicted in Fig. 6.

In general, the GPC algorithm computes the control sequence $\{\Delta u(k+j)\}, j=1, \ldots, H_c$, such that the following quadratic cost function is minimized:

$$J(H_{p1}, H_{p2}, H_c, \lambda) = \sum_{j=H_{p1}}^{H_{p2}} (\omega(k+j) - \hat{\mathbf{y}}(k+j))^2 + \lambda \sum_{j=1}^{H_c} \Delta u^2(k+j-1) \tag{21}$$

Here, $\hat{\mathbf{y}}(k+j)$ denotes the predicted system output, $\omega(k+j)$ the modified setpoint that is assumed to be known in advance, H_{p1} the minimum costing horizon, H_{p2} the maximum costing or prediction horizon, H_c the control horizon, and λ is the move suppression coefficient.

5. Results

The inputs to the FH model are the average temperature, the fuel flow, the air flow, and the current. The output of the FH model is the voltage (y_1). The average temperature is taken as the arithmetic average of that of the cathode inlet and cathode exhaust, under constant air flow.

The linear subsystem in the Hammerstein model is represented using the rational orthonormal bases with fixed poles

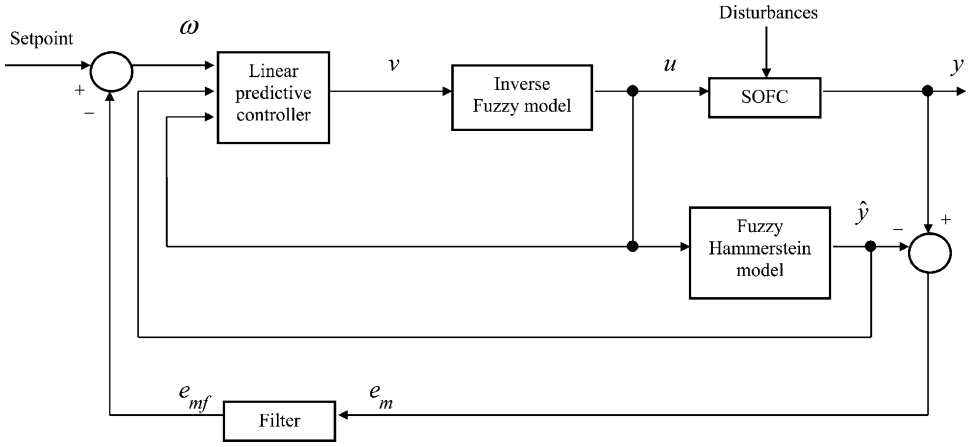

Fig. 6. Fuzzy Hammerstein model based predictive control scheme.

studied in [33,34],

$$\mathscr{B}_l(q) = \left(\frac{\sqrt{1-|\xi_l|^2}}{q-\xi_l}\right)\prod_{i=0}^{l-1}\left(\frac{1-\bar{\xi}_i q}{q-\xi_i}\right), \quad l \geq 1 \tag{22}$$

$$\mathscr{B}_0(q) = \left(\frac{\sqrt{1-|\xi_0|^2}}{q-\xi_0}\right) \tag{23}$$

where $(\xi_0, \xi_1, \ldots, \xi_{p-1})$ are the poles of the bases.

In order to determine the model order of the linear subsystem, as well as initial guesses for the location of the poles of the bases, the same input–output data are used to identify a linear model of the process using a subspace method. System identification toolbox for use with MATLAB [35] is used for the identification of the linear model. As a result of the identification process a fourth order model is estimated as the linear part of the Hammerstein model.

In this paper, the rules of the fuzzy system are designed based on the available a priori knowledge and the parameters of the membership, and the consequent are adapted in a learning process based on the available input–output data.

For a good model performance, the antecedent fuzzy sets on the input variables are designed. Fig. 7 shows the membership functions corresponding to temperature, current and fuel flow. The nominal operating conditions of the column considered in this example are given in Table 1 [36,37].

The amount of fuel flow can be controlled according to the current. The current is proportional to the terminal load. It is evident that an increase in the load growths' current, which in turn decreases output voltage of the fuel cell. The increase in current increases fuel flow rate. This increase increments power flow from the SOFC to the load. Fig. 8a shows the load step changes while Fig. 8b represents the output current due to these changes in the load.

The conjunct variation of output current and air utilization factor, together with the consequent variation of the inlet fuel mass flow, is the principal way to regulate the electric power production. In this paper, SOFC stack operates at constant air flow, and with the air utilization factor changed proportionally to the requested cell current. The variation of fuel flow is depicted in Fig. 9a.

The functioning at low cell current decreases the power production and the stack temperature. An increase of the air utilization factor tends to a higher power generation and to a large increase of the SOFC temperature. The variation of temperature is described in Fig. 9b.

Two different controller configurations are used to illustrate the performance of the SOFC.

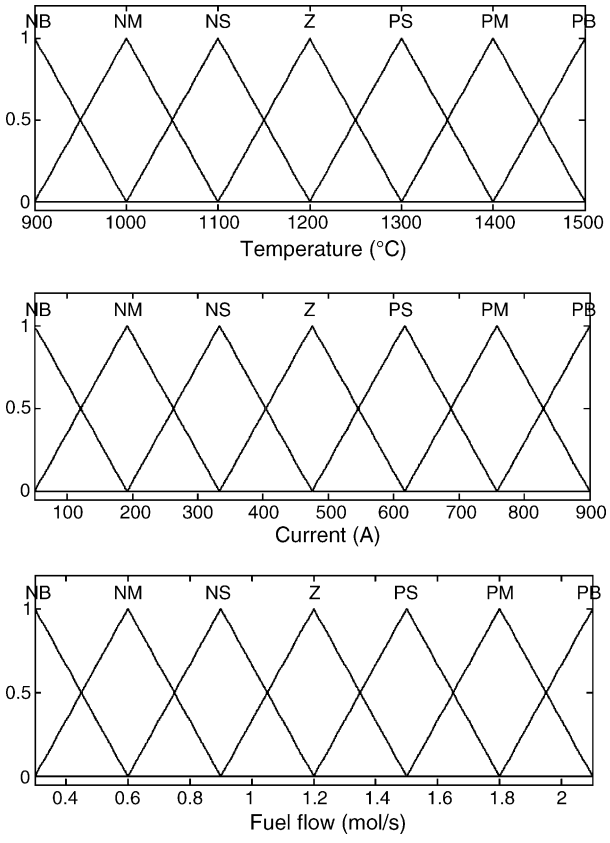

Fig. 7. Antecedent membership functions.

Table 1
Operating point data

Power (kW)	100
Stack voltage (V)	286.3
Stack current (A)	300
Number of cells	384
Number of stacks	1
Open circuit voltage for each cell (V)	0.935
Input fuel flow ($\times 10^{-3}$ kmol s^{-1})	1.2
Input air flow ($\times 10^{-3}$ kmol s^{-1})	2.4
Cell area (cm^2)	1000
Cell temperature (°C)	1000
Transfer coefficient, α	0.5
Ohmic resistance constant, β_r	−2870
Ohmic resistance constant, α_r	0.2
Temperature constant, T^0 (K)	923
Limiting current (A m^{-2})	0.8
K_{H_2} ($\times 10^{-4}$ kmol atm^{-1} s^{-1})	8.43

- Linear MPC using a model obtained by ARX identification of the complete system model at nominal operating conditions [5].
- Fuzzy Hammerstein MPC.

The MATLAB implementation of quadratic programming is used [38]. The MPC parameters are selected according to the tuning rules given in [39]. The two methods use identical control parameters.

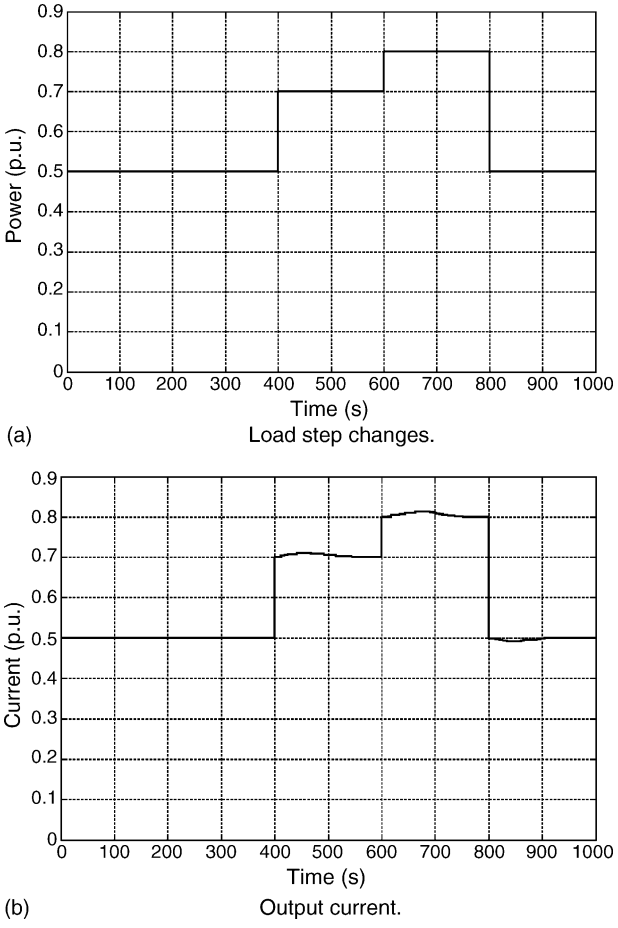

Fig. 8. (a) Load step changes and (b) output current.

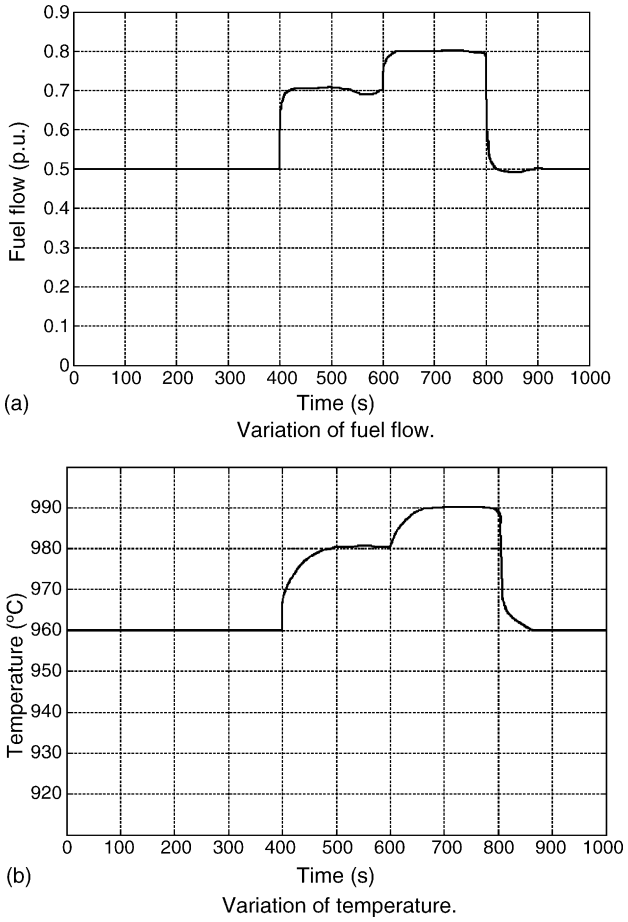

Fig. 9. (a) Variation of fuel flow and (b) variation of temperature.

The focus of the following simulations is to contrast the performance of the MPC framework using the Hammerstein model and the linear model with the complete SOFC model.

The performances using both the linear and Hammerstein models are evaluated with constraints. Fig. 10 shows the output voltage response of the SOFC due to the change in fuel flow input. Comparing the responses, one will notice that the

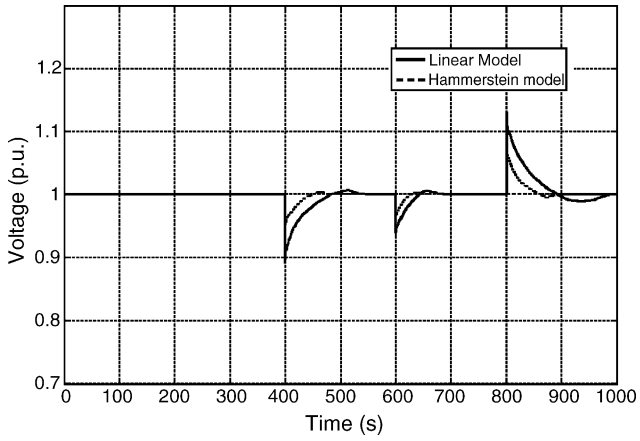

Fig. 10. Output voltage response.

Hammerstein model yields a significantly better closed-loop response.

To evaluate the performance of the control system, the integral of time absolute error (ITAE) will be used:

$$\text{ITAE} = \int_0^{T_f} [\beta|e(t)|t]\,\mathrm{d}t \tag{24}$$

where T_f is a finite time chosen somewhat bigger than the rising time and around the settling time, $e(t)$ is the error which is the difference between the measured output and the set-point which is chosen as a unit step, and β is a scaling factor. The resulting ITAE using linear MPC is 1501.3, which is rather higher than that of using Hammerstein MPC (352.6).

6. Conclusions

The Hammerstein model is a special kind of nonlinear systems. This paper proposes a multivariable fuzzy Hammerstein (FH) model of a SOFC.

This work has described a model based controller (MPC) for the regulation of a SOFC. A MPC based on a FH model has been developed, enabling the use of optimal control to satisfy power demands.

It is clear that the success of MPC is highly dependent on having a reliable SOFC model. It is very important to look for a model that may effectively describe the nonlinear behavior of the SOFC, and must also be easily usable in designing a MPC algorithm.

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